The Origin of Mass of the visible Universe

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what is the source of the mass of ordinary matter?

(lattice talk: controlling systematics)
Outline

1. Introduction
2. QCD as a theory
3. Lattice QCD
4. Mass of the proton
5. Summary
The origin of mass of the visible Universe

source of the mass for ordinary matter (not a dark matter talk)

basic goal of LHC (Large Hadron Collider, Geneva Switzerland):

“to clarify the origin of mass”

e.g. by finding the Higgs particle, or by alternative mechanisms

order of magnitudes: 27 km tunnel and $O(10)$ billion dollars
The vast majority of the mass of ordinary matter

ultimate (Higgs or alternative) mechanism: responsible for the mass of the leptons and for the mass of the quarks

interestingly enough: just a tiny fraction of the visible mass (such as stars, the earth, the audience, atoms) electron: almost massless, $\approx 1/2000$ of the mass of a proton quarks (in ordinary matter): also almost massless particles

the vast majority (about 95%) comes through another mechanism $\Rightarrow$ this mechanism and this 95% will be the main topic of this talk quantum chromodynamics (QCD, strong interaction) on the lattice
The mass is not the sum of the constituents’ mass

usually the mass of “some ordinary thing” is just the sum of the mass of its constituents (upto tiny corrections)

origin of the mass of the visible universe: dramatically different
proton is made up of massless gluons and almost massless quarks

the mass of a quark is ≈5 MeV, that of the proton is ≈1000 MeV
hadron masses (and other questions) many results in the literature

JLQCD, PACS-SC (Japan), MILC (USA), QCDSF (Germany-UK), RBC & UKQCD (USA-UK), ETM (Europe), Alpha (Europe) JLAB (USA), CERN-Rome (Swiss-Italian)

note, that all of them neglected one or more of the ingredients required for controlling all systematics (it is quite CPU-demanding)

⇒ Budapest-Marseille-Wuppertal (BMW) Collaboration
DEISA partner supercomputers: Juelich (Jugene), and CNRS (IDRIS)

try to control all systematics: Science 322:1224-1227,2008

http://www.bmw.uni-wuppertal.de
Proton’s Mass ‘Predicted’

Starting from a theoretical description of its innards, physicists precisely calculated the mass of the proton and other particles made of quarks and gluons. The numbers aren’t new; experimenters have been able to weigh the proton for nearly a century. But the new results show that physicists can at least make accurate calculations of the ultracomplex strong force that binds quarks.

In simplest terms, the proton comprises three quarks with gluons zipping between them to convey the strong force. Thanks to the uncertainties of quantum mechanics, however, myriad gluons and quark-antiquark pairs flit into and out of existence within a proton in a frenzy that’s nearly impossible to analyze but that produces 95% of the particle’s mass.

To simplify matters, theorists from France, Germany, and Hungary took an approach known as “lattice quantum chromodynamics.” They modeled continuous space and time as a four-dimensional array of points—the lattice—and confined the quarks to the points and the gluons to the links between them. Using supercomputers, they reckoned the masses of the proton and other particles to a precision of about 2%—a tenth of the uncertainties a decade ago—as they reported in November.

In 2003, others reported equally precise calculations of more-esoteric quantities. But by calculating the familiar proton mass, the new work signals more broadly that physicists finally have a handle on the strong force.
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Introduction to particle physics

Basic idea of particle physics: quantum field theory

classical field theory:
variables depend on the space- and time-coordinates: \((\vec{r}, t)\)

meteorology: temperature \(T(\vec{r}, t)\), pressure \(p(\vec{r}, t)\), wind \(\vec{v}(\vec{r}, t)\).
electromagnetism: \(\vec{E}(\vec{r}, t), \vec{B}(\vec{r}, t)\).

quantum theory:
new phenomenon, measurables are not commuting:
ordinary numbers are commuting \(5 \times 6 = 6 \times 5\)
but there are non-commuting examples even in everyday life

in quantum theory: e.g. \(\hat{x}\hat{v} \neq \hat{v}\hat{x}\)
(position and speed in quantum mechanics)
Oscillator

classical mechanics: position oscillates, energy is arbitrary (it takes continuous values)

classical field theory: fields oscillate, \( \vec{E}(\vec{r}, t), \vec{B}(\vec{r}, t) \)

quantum mechanics: only some fixed energy levels \( E_i \) are allowed

quantum field theory: \( E_i \) packages move in space as \( t \) goes on

\( \rightarrow \) particle interpretation (e.g. photons, electrons)
Symmetries

particle physics reflects the symmetries around us

the world is full of symmetries: right hand vs. left hand
we drive on the right, but we could drive on the left
(actually billions of people do that)

if we turn around we see a different world (looks like that)
same world: distances between any two points remained the same

rotations are described by 2*2 or 3*3 matrices (tables)

the same symmetry (3*3) in quantum chromo dynamics (QCD)
rotate the field variables by 3*3 matrices ⇒ same solutions
(space dependent: gauge symmetry & gauge theory)
Left-right symmetry in everyday life
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Chromodynamics

it is an electromagnetism-like theory with three charges

electromagnetism: one charge, it can be positive/negative
degrees of freedom: $e^+/e^-$, photons transmit the interaction

ordinary matter is usually neutral (non-charged)
screened charges play an important role: van der Waals force

chromodynamics: three different types of charges
we call them colors: blue, red, green (not real colors)
all three can be positive/negative
degrees of freedom: quarks/antiquarks, gluons (like photons)

the forces are much stronger than in electromagnetism
hence the name: strong interaction
ordinary matter is color neutral (otherwise forces would be huge)
screened charges play an important role: nuclear bomb, sun
Confinement

for a single color charge the forces would be huge
⇒ color charge is neutralized by other color charges
we can not separate them: confinement phenomenon

how to realize color neutral combinations: bound state hadrons

first possibility: as in electromagnetism
positive and negative charges together: red+antired=white
mesons ($\pi$, K, $\rho$ ...) are quark-antiquark bound states

second possibility: as in color television
3 base-colors lead to color neutrality: red+blue+green=white
protons, neutrons are quark-quark-quark bound states
(we call them baryons p, n, $\Lambda$, $\Sigma$, $\Xi$ ...)
electrodynamics: electromagnetic field is given by $A$, electron by $\psi$

$$-\frac{1}{4g^2} F_{\mu\nu} F^{\mu\nu} + \bar{\psi} \left[ i \gamma_{\mu} (\partial_{\mu} + iA_{\mu}) + m \right] \psi, \quad F_{\mu\nu} = \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu}$$

chromodynamics: field $A_{\mu}$ is a traceless 3*3 matrix, $\psi$ has and index

$$-\frac{1}{4g^2} \text{tr} F_{\mu\nu} F^{\mu\nu} + \bar{\psi} \left\{ i \gamma_{\mu} (\partial_{\mu} + iA_{\mu}) + m \right\} \psi, \quad F_{\mu\nu} = \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu} + i[A_{\mu} A_{\nu} - A_{\nu} A_{\mu}]$$

gauge invariance unambiguously fixes this form

this is the classical level of the field theory, we quantize it strongly interacting theory: difficult to solve
Degrees of freedom

even worse: no sign of the same physical content

Lagrangian contains massless gluons & almost massless quarks
we detect none of them, they are confined
we detect instead composite particles: protons, pions

proton is several hundred times heavier than the quarks

how and when was the mass generated

qualitative picture (contains many essential features):
in the early universe/heavy ion experiment: very high temperatures (motion)

it is diluted by the expansion (of the universe/experimental setup)

small fraction remained with us confined in protons

⇒ the kinetic energy inside the proton gives the mass \( E = mc^2 \)
systematic non-perturbative approach (numerical solution):

quantum fields on the lattice

quantum theory: path integral formulation with $S=E_{\text{kin}}-E_{\text{pot}}$

quantum mechanics: for all possible paths add $\exp(iS)$
quantum fields: for all possible field configurations add $\exp(iS)$

Euclidean space-time ($t=i\tau$): $\exp(-S)$ sum of Boltzmann factors

we do not have infinitely large computers $\Rightarrow$ two consequences

a. put it on a space-time grid (proper approach: asymptotic freedom)
formally: four-dimensional statistical system
b. finite size of the system (can be also controlled)

$\Rightarrow$ stochastic approach, with reasonable spacing/size: solvable
fine lattice to resolve the structure of the proton (≤0.1 fm) few fm size is needed 50-100 points in ‘xyzt’ directions $a \Rightarrow a/2$ means 100-200 × CPU mathematically

$10^9$ dimensional integrals advanced techniques, good balance and several Tflops are needed
Historical background

1972 Lagrangian of QCD (H. Fritzsch, M. Gell-Mann, H. Leutwyler)

   at small distances (large energies) the theory is “free”

1974 lattice formulation (Kenneth Wilson)
   at large distances the coupling is large: non-perturbative

Nobel Prize 2008: Y. Nambu, & M. Kobayashi T. Masakawa

spontaneous symmetry breaking in quantum field theory
   strong interaction picture: mass gap is the mass of the nucleon
mass eigenstates and weak eigenstates are different
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Scientific Background on the Nobel Prize in Physics 2008

“Even though QCD is the correct theory for the strong interactions, it can not be used to compute at all energy and momentum scales ... (there is) ... a region where perturbative methods do not work for QCD.”

true, but the situation is somewhat better: new era fully controlled non-perturbative approach works (took 35 years)
Importance sampling

\[ Z = \int \prod_{n, \mu} [dU_\mu(n)] e^{-S_g} \det(M[U]) \]

we do not take into account all possible gauge configuration

each of them is generated with a probability \( \propto \) its weight

Metropolis step for importance sampling:
(all other algorithms are based on importance sampling)

\[ P(U \to U') = \min \left[ 1, \exp(-\Delta S_g) \frac{\det(M[U'])}{\det(M[U])} \right] \]

gauge part: trace of 3\(\times\)3 matrices (easy, without \(M\): quenched)
fermionic part: determinant of 10\(^6\) \(\times\) 10\(^6\) sparse matrices (hard)

more efficient ways than direct evaluation (\(Mx=a\)), but still hard
Quenched results

QCD is 35 years old ⇒ properties of hadrons (Rosenfeld table)

non-perturbative lattice formulation (Wilson) immediately appeared needed 20 years even for quenched result of the spectrum (cheap) instead of det(M) of a $10^6 \times 10^6$ matrix trace of $3 \times 3$ matrices

always at the frontiers of computer technology:

GF11: IBM "to verify quantum chromodynamics" (10 Gflops, ’92)
CP-PACS Japanese purpose made machine (Hitachi 614 Gflops, ’96)

the $\approx 10\%$ discrepancy was believed to be a quenching effect
Ingredients to control systematics

- inclusion of det[M] with an exact $n_f=2+1$ algorithm action: universality class is known to be QCD (Wilson-quarks)
- spectrum for the light mesons, octet and decuplet baryons (three of these fix the averaged $m_{ud}$, $m_s$ and the cutoff)
- large volumes to guarantee small finite-size effects
  - rule of thumb: $M_\pi L \gtrsim 4$ is usually used (correct for that)
- controlled interpolations & extrapolations to physical $m_s$ and $m_{ud}$
  - or eventually simulating directly at these masses
  - since $M_\pi \approx 135$ MeV extrapolations for $m_{ud}$ are difficult
  - CPU-intensive calculations with $M_\pi$ reaching down to $\approx 200$ MeV
- controlled extrapolations to the continuum limit ($a \to 0$)
  - calculations are performed at no less than 3 lattice spacings
Simulation at physical quark masses

M's eigenvalues close to 0: CPU demanding (large condition number) our choice of action and large volumes (6 fm): the spread of the smallest eigenvalue decreases ⇒ away from zero

we can go down to physical pion masses ⇒ algorithmically safe

Blue Gene shows perfect strong scaling from 1 midplane to 16 racks our sustained performance is as high as 37% of the peak 0.2 Pflops
altogether 15 points for each hadrons

smooth extrapolation to the physical pion mass (or $m_{ud}$)
small discretization effects (three lines barely distinguishable)

continuum extrapolation goes as $c \cdot a^n$ and it depends on the action
in principle many ways to discretize (derivative by 2,3... points)
goal: have large $n$ and small $c$ (in our case $n = 2$ and $c$ is small)
Final result for the hadron spectrum

Budapest-Marseille-Wuppertal collaboration

- $\pi$
- $K$
- $K^*$
- $\rho$
- $N$
- $\Lambda$
- $\Sigma$
- $\Sigma^*$
- $\Omega$
- $\Xi$
- $\Xi^*$
- $\Delta$

- experiment
- width
- input
- QCD

Z. Fodor

The Origin of Mass of the visible Universe
understanding the source and the course of the mass generation of ordinary matter is of fundamental importance

after 35 years of work these questions can be answered (cumulative improvements of algorithms and machines are huge)

they belong to the largest computational projects on record

perfect tool to understand hadronic processes (strong interaction)